# Computational methods in applied inverse problems

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**Abstract**

Below is a brief description of my planned short course at IMPA, given as part of the Thematic Program on Parameter Identification in Mathematical Models. It consists of four lectures, at most 90 minutes each, planned for October 17, 19, 24 and 26, 2017.

In the past two decades there have been many developments in computational methods for applied inverse problems. These include PDE constrained optimization, sparsity-enhancing methods, level set methods, probabilistic methods, randomized algorithms, machine learning techniques (e.g., deep learning) and more. Optimization techniques play a prominent role, as do PDE discretization methods and fast solution techniques. I will attemp to shed some light on several of the challenges and solution techniques in these computational areas, using my own research to demonstrate and highlight issues.

This document is meant to describe a tentative rather than final plan. The lectures will be adjusted according to audience level of interest and needs as well as the instructor's limitations.

## 1 Calibration and simulation of deformable objects.

I will discuss and demonstrate methods, issues, and results in a computer graphics setting [8, 2]. These lead to rich applied mathematics and numerical methods endeavours. Then I'll discuss several things that were potentially missing in the graphics works and highlight success alongside with persistent issues.

This leads to

1. PDE/ODE systems and simulation methods for deformable bodies

2. Motion tracking (a machine learning-type inverse problem)

3. Parameter estimation in hyperbolic-type PDEs

4. Handling data

Some of these issues are further desribed in this lecture, while others are delayed to future ones.

## 2 Data manipulation and completion

Consider inverse problems whose forward operator involves the solution of a PDE system. The PDE depends on some material property (a distributed parameter that forms a surface over the PDE domain), and the purpose of the inverse problem is to calibrate the model by estimating the distributed parameter function. This is done by requiring a given function of the field to match a set of given noisy measured data. Often in applications the data is available only at a restricted set of locations, or situations, while existence and uniqueness theory, or other considerations, demand that a fuller set (e.g., "data everywhere") be given. It is then tempting to *complete the data*, e.g., by a potentially sophisticated interpolation, before starting the inverse problem solution process. Such data completion, however, has its well-known perils as well.

This lecture describes our various techniques for handling (or avoidance) of data completion in the context of practical applications that include

1. Electromagnetic data inversion in geophysical exploration [3]

2. Local volatility surface calibration for commodity markets in finance [1]

3. Plant motion tracking and calibration in computer graphics [8]

4. Monte Carlo methods for problems involving many data sets (or experiments) [6]

## 3 Estimating the trace of a large implicit matrix and applications

This lecture describes algorithms, theoretical results and practical application of work that is mainly due to my former PhD student Fred Roosta, with participation by Kees van den Doel and myself [4, 7, 5].

Inverse problems involving systems of PDEs can be very expensive to solve numerically. This is so especially when many experiments, involving different combinations of sources and receivers, are employed in order to obtain reconstructions of acceptable quality. The mere evaluation of a misfit function (the distance between predicted and observed data) often requires thousands of PDE solves. We develop and assess randomized algorithms for dimensionality reduction, to make the corresponding computational burden tolerable.

The essence of such algorithms boils down to estimating the least squares misfit function, which in turns leads to Monte-Carlo methods for trace estimation for implicit matrices. We state and prove theoretical probabilistic bounds regarding the efficiency of such methods, depending on the probability distribution and the matrix properties.

The approach and algorithms are demostrated on the DC resistivity problem with rough solutions as well as other problems. Highly efficient variants of the resulting algorithms are identified.

## 4 Numerical Analysis and Visual Computing: not too little, not too much

Visual computing is a wide area that includes computer graphics and image processing applications, where the "eyeball-norm" rules.

I will discuss two case studies involving numerical methods and analysis applied to this area.

1. The first case study involves motion simulation and calibration of soft objects such as cloth, plants and skin. The lecture continues Lecture 1 and uses material from current joint work with Edwin Chen and Dinesh Pai. The governing elastodynamics PDE system, discretized in space already at the variational level using FEM, leads to a large, expensive to assemble, dynamical system in time, where the damped motion may mask highly oscillatory stiffness. Geometric integration ideas are making their way into visual computing research these days in search for more quantitative computations.

2. The other case study involves some image processing problems where there is a premium for local approaches that do not necessarily involve PDEs. I will demonstrate and discuss.

## References


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